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Enhancement of photon antibunching by passive interferometry

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Photon antibunching in intracavity second-harmonic generation and intracavity atomic absorption inside a high-Q optical cavity is discussed. A linearized description of quantum dynamics in the weak-field limit is used because the system size, as characterized by the number of photons n_0 needed to probe the nonlinearity of the system, is often quite large for realistic parameters. In both cases standard results are recalled that demonstrate that antibunching is a small effect given by $g^{(2)}(0)-1\approx-1/n_0$ for large values of n_0. We show that when higher-order terms that are usually ignored in linearized treatments are retained, $g^{(2)}(0)$ satisfies the lower bound $g^{(2)}(0)\geq 0$ even for small values of n_0. It is further shown that by employing a high-Q cavity to suppress the coherent part of the spectrum of field fluctuations, perfect antibunching can be achieved even for large systems. Conditions under which this is possible are derived and curves are presented to illustrate the behavior.

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I. INTRODUCTION

Many nonlinear dissipative systems in optical physics involving, for example, the atom-field interaction or the parametric interaction between different modes of a cavity exhibit nonclassical features [1–6]. These features are conveniently described in terms of the statistical properties of the electromagnetic field. Unfortunately, most of these systems are such that many photons are needed to explore the nonlinearity of the system so that system dynamics are such that a very small quantum noise evolves around a classical steady state. The smallness of quantum noise makes a linearized treatment of the quantum dynamics of the system possible, with quantum effects inherently small compared to the size of the classical steady state. Examples of case in point are photon antibunching effects predicted in second-harmonic generation [1,3] and multi atom optical bistability [4–6]. In both cases the antibunching effect is inversely proportional to the size of the system as characterized by some parameter n_0. For multiatom bistability in the good-cavity limit this parameter is the saturation photon number, and for intracavity second-harmonic generation it is the threshold photon number. In principle, at least, it is possible to enhance the relative size of quantum effects by reducing the system size. In optical bistability, for example, the system-size parameter n_0 (the saturation photon number) was of the order of 10^3 for the work of Ref. [7]. The corresponding antibunching effect is then predicted to be considerably less than 1%. By searching for appropriate atomic species (Cs, for example) and working with short (~ 1 mm in length) optical cavities of small mode volume, the saturation photon number can be reduced down to below unity. By further utilizing a high-finesse ($\mathcal{F}\geq 10^4$) cavity and employing an atomic cooperativity parameter $C\sim 20$, the antibunching effect can be increased [6] to about 20% as has been observed in recent experiments [17]. The corresponding problem of reducing the system size in second-harmonic generation is more formidable [3]. For most experimental situations the threshold photon number is of the order of 10^6–10^{-8}. The corresponding antibunching effect is of the order of 10^{-6}–10^{-8}. By using high-finesse optical cavities and crystals with a large nonlinear coefficient, the threshold photon number can perhaps be lowered to 10^3-10^4. The resulting antibunching effect (of order 10^{-3}–10^{-4}) is still woefully small. Thus the approach of system-size reduction is not always feasible. Furthermore, with a reduction in system size the simplified linearized treatment of quantum dynamics may be rendered invalid. From the experimental point of view then the general problem we face is this: how do we enhance and therefore measure a small quantum effect riding on a strong classical (coherent) background. One approach to this problem, based on an interference experiment, was discussed by Bandilla and Ritze [8]. Here we explore yet another approach [9], which employs a passive filter cavity, external to the system producing quantum effects, to provide a variable for the classical (coherent) carrier while leaving the spectrum of quantum fluctuations largely intact, thereby enhancing otherwise small quantum effects. The quantum effects of interest to us throughout this paper are photon antibunching and sub-Poissonian photon statistics [3,5]. The fields considered in this paper also exhibit squeezing. In general, however, squeezing and antibunching refer to different nonclassical properties of the electromagnetic field. We are not concerned here with squeezing of the fields, although the method of this paper can also be applied to a discussion of squeezing. In Sec. II we present a general approach to the problem of filtering by a high-Q optical cavity. Section III applies these results to antibunching and sub-Poissonian photon statistics in intracavity second-harmonic generation, and Sec. IV considers the same effects in intracavity absorption by a set of two-level atoms. Principal results and conclusions of this paper are presented in Sec. V.

II. PASSIVE CAVITY FILTER

Consider a passive Fabry-Pérot-type high-Q optical cavity consisting of two mirrors with field decay rates κ_1 and κ_2 as shown in Fig. 1. The total decay rate for protons in the cavity is $2(\kappa_1+\kappa_2)$. Because of the finite transitivity of the mirrors, the cavity field is coupled to the fields outside. We are interested in narrow-band fields centered at frequency ω_0 such that the variations of mirror reflectivities across the bandwidths of the fields involved can be ignored. Let $\hat{c}$ and $\hat{c}^\dagger$ denote the annihilation and creation operators for the field inside the cavity. Then a quantum-mechanically consistent application of boundary conditions at the mirrors requires a coupling of the intracavity field operator $\hat{c}$ to operators $\hat{c}_{\text{inc}}$, $\hat{c}_{\text{ref}}$, $\hat{d}_{\text{inc}}$, and $\hat{d}_{\text{trans}}$ representing fields outside the filter cavity as shown in Fig. 1. The equation of motion for $\hat{c}$ in the interaction picture is [10]

$$\frac{d\hat{c}}{dt}=-(\kappa_1+\kappa_2)\hat{c}+\sqrt{2\kappa_1}\hat{c}_{\text{inc}}+\sqrt{2\kappa_2}\hat{d}_{\text{inc}} , \tag{1}$$

where $\hat{c}_{\text{inc}}(t)$ and $\hat{d}_{\text{inc}}(t)$ represent operators for the field incident from the left and right of the filter cavity, respectively, and $\hat{c}_{\text{ref}}(t)$ and $\hat{d}_{\text{trans}}(t)$ represent the fields leaving the cavity (reflected or transmitted fields). In our case $\hat{c}_{\text{inc}}(t)$ would correspond to the field $\hat{a}_{\text{out}}$ radiated by the system cavity, $\hat{c}_{\text{ref}}(t)$ represents the reflected field, $\hat{d}_{\text{inc}}(t)$ represents the vacuum field (no light is incident from the right), and $\hat{d}_{\text{trans}}(t)$ represents the transmitted field. Note that the internal field operator $\hat{c}(t)$ has dimensions such that $\hat{c}^\dagger(t)\hat{c}(t)$ is the photon-number operator for the intracavity field, and the operators for the fields outside have dimensions such that $\hat{c}^\dagger_{\text{inc}}(t)\hat{c}_{\text{inc}}(t)$ represents a photon-number flux (photons/sec) operator, for example. The application of boundary conditions is carried out most conveniently in terms of the Fourier components of the field. In order to introduce a Fourier decomposition of an annihilation operator we recall that the annihilation operator contains only positive-frequency components [11]. If we write any one of the various annihilation operators in the form $\hat{A}(t)e^{-i\omega_0 t}$, where ω_0 is some suitably chosen carrier frequency, then the Fourier decomposition of $\hat{A}(t)$ can be written as

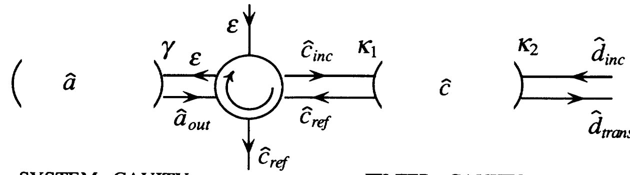

FIG. 1. An outline of the arrangement to enhance antibunching.

$$\begin{aligned}\hat{A}(t)&=e^{i\omega_0 t}\frac{1}{2\pi}\int_0^\infty d\omega\, \hat{A}(\omega)e^{-i\omega t}\\ &=\frac{1}{2\pi}\int_{-\omega_0}^\infty d\Omega\, \hat{\hat{A}}(\omega_0+\Omega)e^{-i\Omega t}\\ &\approx\frac{1}{2\pi}\int_{-\infty}^\infty d\Omega\, \hat{\hat{A}}(\Omega)e^{-i\Omega t} ,\end{aligned} \tag{2}$$

where $\Omega=\omega-\omega_0$ and for notational convenience we have written $\hat{\hat{A}}(\Omega)$ for $\hat{\hat{A}}(\Omega+\omega_0)$. The lower limit of integration in Eq. (2) has been extended from $-\omega_0$ to $-\infty$. For narrow-band optical fields, such as those considered in this paper, this introduces negligible error and results in great simplicity of calculation and notation. We choose frequency ω_0 to be the resonance frequency of the filter cavity so that all operators appearing in Eq. (1) may be considered interaction-picture operators. Assuming all inputs to the filter cavity are in resonance with the cavity frequency and using the Fourier decomposition (2) for each of the operators in Eq. (1), we obtain the spectral component of the intracavity field of the filter cavity in terms of the spectral components of the external fields as [10]

$$\hat{\hat{c}}(\Omega)=\frac{\sqrt{2\kappa_1}\hat{\hat{c}}_{\text{inc}}(\Omega)+\sqrt{2\kappa_2}\hat{\hat{d}}_{\text{inc}}(\Omega)}{\kappa_1+\kappa_2-i\Omega} . \tag{3}$$

The reflected field spectral component $\hat{\hat{c}}_{\text{ref}}(\Omega)$ is then obtained by using the boundary conditions at mirror 1 which leads to

$$\begin{aligned}\hat{\hat{c}}_{\text{ref}}(\Omega)&=\sqrt{2\kappa_1}\hat{\hat{c}}(\Omega)-\hat{\hat{c}}_{\text{inc}}(\Omega)\\ &=\frac{(\kappa_1-\kappa_2+i\Omega)\hat{\hat{c}}_{\text{inc}}(\Omega)+2\sqrt{\kappa_1\kappa_2}\hat{\hat{d}}_{\text{inc}}(\Omega)}{\kappa_1+\kappa_2-i\Omega} .\end{aligned} \tag{4}$$

Similarly, the boundary condition at mirror 2 gives the transmitted field component

$$\hat{\hat{d}}_{\text{trans}}(\Omega)=\frac{(\kappa_2-\kappa_1+i\Omega)\hat{\hat{d}}_{\text{inc}}(\Omega)+2\sqrt{\kappa_1\kappa_2}\hat{\hat{c}}_{\text{inc}}(\Omega)}{\kappa_1+\kappa_2-i\Omega} . \tag{5}$$

Apart from the presence of vacuum field operators $\hat{\hat{d}}_{\text{inc}}$, Eqs. (4) and (5) are the same as the classical equations for the reflected and transmitted fields from a cavity [12]. Quantum-mechanical consistency (preservation of commutation rules) requires the presence of terms involving $\hat{\hat{d}}_{\text{inc}}$ even if the corresponding fields are in the vacuum state (no light input). Equations (4) and (5) express the reflected and transmitted fields in terms of the fields incident on the cavity and are the basic equations of this paper. Note that in practice a unidirectional circulator would be placed between the system and filter cavities for the standing-wave cavities shown in Fig. 1. The output of the system cavity $\hat{a}_{\text{out}}$ would then become the input

$\hat{c}_{\rm inc}$ to the filter cavity and the output $\hat{c}_{\rm ref}$ would be steered to a detection system.

Using Eqs. (4) and (5) we can relate the spectral densities of the field reflected from the filter cavity and the field incident from the system cavity. The 2×2 spectral density matrix $S(\Omega)$ of a field associated with an annihilation operator $\hat{A}(t)$ is related to the normally ordered spectral correlation functions by

$$S(\Omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\Omega'\begin{bmatrix}\langle:\hat{A}(\Omega)\hat{A}(\Omega'):\rangle & \langle:\hat{A}^\dagger(\Omega)\hat{A}(\Omega'):\rangle\\ \langle:\hat{A}^\dagger(\Omega)\hat{A}(\Omega'):\rangle & \langle:\hat{A}^\dagger(\Omega)\hat{A}^\dagger(\Omega'):\rangle\end{bmatrix}. \tag{6}$$

For stationary fields the spectral correlation matrix can be written in the form

$$\begin{bmatrix}\langle:\hat{A}(\Omega)\hat{A}(\Omega'):\rangle & \langle:\hat{A}^\dagger(\Omega)\hat{A}(\Omega'):\rangle\\ \langle:\hat{A}^\dagger(\Omega)\hat{A}(\Omega'):\rangle & \langle:\hat{A}^\dagger(\Omega)\hat{A}^\dagger(\Omega'):\rangle\end{bmatrix}=2\pi\begin{bmatrix}S_{11}(\Omega)\delta(\Omega+\Omega') & S_{12}(\Omega)\delta(\Omega-\Omega')\\ S_{12}(\Omega)\delta(\Omega-\Omega') & S_{22}(\Omega)\delta(\Omega+\Omega')\end{bmatrix}, \tag{7}$$

where $S_{ij}(\Omega)$ are the matrix elements of the spectral density matrix $S(\Omega)$. In Secs. III and IV we will have occasions to decompose the spectral-density matrix into two parts, $S(\Omega)=S^0(\Omega)+\delta S(\Omega)$, with matrix elements given by

$$S_{ij}(\Omega)=2\pi S_{ij}^0\delta(\Omega)+\delta S_{ij}(\Omega). \tag{8}$$

Here the constant matrix S^0 represents the coherent part of the spectrum which is proportional to a δ function centered at $\Omega=0$ and $\delta S_{ij}(\Omega)$ represents the incoherent part of the spectrum. The spectral density $S^{\rm ref}(\Omega)$ of the field reflected from the filter cavity is now easily found by using Eqs. (4), (6), and (7) and the fact that for normally ordered averages, the vacuum field $\hat{d}_{\rm inc}$ makes no contribution. This gives the following relation between the spectral density $S^{\rm ref}(\Omega)$ of the field reflected from the filter cavity and the spectral density $S^{\rm inc}(\Omega)$ of the field incident on the filter cavity,

$$S^{\rm ref}(\Omega)=\frac{(\kappa_1-\kappa_2)^2+\Omega^2}{(\kappa_1+\kappa_2)^2+\Omega^2}S^{\rm inc}(\Omega). \tag{9}$$

Apart from quantum-mechanical averages involved in the calculation of $S(\Omega)$ this equation is identical to the corresponding equation in classical optics.

Once the spectral density matrix $S(\Omega)$ of any of the various fields is known, the two-time field correlation matrix can be calculated by inverse Fourier transform since the spectral density matrix is related to the steady-state correlation matrix $\Gamma(\tau)$ by

$$\Gamma(\tau)\equiv\begin{bmatrix}\langle:\hat{A}(\tau)\hat{A}(0):\rangle & \langle:\hat{A}^\dagger(\tau)\hat{A}(0):\rangle\\ \langle:\hat{A}^\dagger(\tau)\hat{A}(0):\rangle & \langle:\hat{A}^\dagger(\tau)\hat{A}^\dagger(0):\rangle\end{bmatrix}=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\Omega\, e^{-i\Omega\tau}S(\Omega). \tag{10}$$

Corresponding to Eq. (8) we have the decomposition

$$\Gamma_{ij}(\tau)=\Gamma_{ij}^0+\delta\Gamma_{ij}(\tau) \tag{11}$$

for the correlation matrix. Here the constant matrix Γ^0 represents the deterministic (coherent) part and $\delta\Gamma(\tau)$ represents the fluctuation part of the time correlation matrix. From Eqs. (8) and (10) it follows that $S_{ij}^0=\Gamma_{ij}^0$. Using Eq. (9) in Eq. (10) we find that the time correlation matrix of the field reflected from the filter cavity can be expressed in terms of the spectral matrix of the input field by

$$\Gamma^{\rm ref}(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\Omega\, e^{-i\Omega\tau}\frac{(\kappa_1-\kappa_2)^2+\Omega^2}{(\kappa_1+\kappa_2)^2+\Omega^2}S^{\rm inc}(\Omega). \tag{12}$$

In the linearized treatment of quantum dynamics we encounter field variables that have Gaussian character in the sense that the correlation functions of all order can be expressed in terms of the second-order correlation function $\Gamma(\tau)$. Equations (9) and (12) thus form the basis of our scheme for the enhancement of antibunching discussed in the next two sections even though higher-order correlation functions are involved. Correlations of the transmitted field can be calculated similarly. These are not of interest to us here and will not be discussed further.

III. ANTIBUNCHING IN INTRACAVITY SECOND-HARMONIC GENERATION

For the system cavity we first consider a high-Q optical cavity where cavity modes at frequency ω_0 (fundamental) and $2\omega_0$ (second harmonic) resonantly interact by way of a nonlinear crystal. The cavity linewidth at the fundamental frequency is γ and at the second-harmonic frequency is γ_2. The fundamental mode is excited by an injected coherent field of normalized amplitude $\mathcal{E}$. The frequency of the pump mode is assumed to be the same as that of the fundamental cavity mode ω_0. By a suitable choice of phases, $\mathcal{E}$ can be taken to be real. The equation of motion for the density matrix operator in the interaction picture is [3]

$$\frac{d\hat{\rho}}{dt}=-\frac{i}{\hbar}[\hat{H}_I,\hat{\rho}]+(\hat{\Lambda}_1+\hat{\Lambda}_2+\hat{\Lambda}_{\rm pump})\hat{\rho}, \tag{13}$$

where the interaction Hamiltonian $\hat{H}_I$ can be written in terms of the annihilation and creation operators $\hat{a}_1(\hat{a}_2)$ and $\hat{a}_1^\dagger\,(\hat{a}_2^\dagger)$ of the fundamental (second-harmonic) mode as

$$\hat{H}_I=\frac{i\hbar g}{2}(\hat{a}_1^{\dagger 2}\hat{a}_2-\hat{a}_1^2\hat{a}_2^\dagger) . \quad (14)$$

The mode coupling constant g for phase-matched operation in the plane-wave approximation is given by $g=2d(\hbar\omega_0^3/\epsilon_0 V_Q)^{1/2}$, where d is the effective nonlinear coefficient, V_Q is the cavity volume, and where we have assumed for simplicity that the nonlinear medium fills the whole cavity. The decay terms describing dissipation of the fundamental mode ($\hat{\Lambda}_1$) and harmonic mode ($\hat{\Lambda}_2$) and the pump term describing excitation are given by

$$\hat{\Lambda}_1\hat{\rho}=\gamma([\hat{a}_1\hat{\rho},\hat{a}_1^\dagger]+[\hat{a}_1,\hat{\rho}\hat{a}_1^\dagger]) , \quad (15)$$

$$\hat{\Lambda}_2\hat{\rho}=\gamma_2([\hat{a}_2\hat{\rho},\hat{a}_2^\dagger]+[\hat{a}_2,\hat{\rho}\hat{a}_2^\dagger]) , \quad (16)$$

$$\hat{\Lambda}_{\text{pump}}\hat{\rho}=\gamma\mathcal{E}([\hat{a}_1^\dagger,\hat{\rho}]-[\hat{a}_1,\hat{\rho}]) . \quad (17)$$

The operator density-matrix equation can be converted into a c-number equation by using one of the coherent-state representations of $\hat{\rho}$. The usual diagonal representation [13] leads to a Fokker-Planck-type equation with a nonpositive diffusion matrix. This approach therefore does not allow us to map Eq. (13) onto a set of c-number stochastic differential equations. This difficulty can be avoided by the use of the so-called positive-P representation [14]. This procedure maps the annihilation and creation operators of the field onto a set of c-number variables according to the rule $(\hat{a}_i,\hat{a}_i^\dagger)\leftrightarrow(\alpha_i,\alpha_{i*})$, where α_i and α_{i*} are two complex variables. Unlike the diagonal representation α_{i*} is not the complex conjugate of α_i except in the mean. Equations of motion for α_i and α_{i*} are [3]

$$\dot{\alpha}_1=-\gamma(\alpha_1-\mathcal{E})+g\alpha_{1*}\alpha_2+\sqrt{\kappa\alpha_2}\xi(t) , \quad (18)$$

$$\dot{\alpha}_{1*}=-\gamma(\alpha_{1*}-\mathcal{E})+g\alpha_1\alpha_{2*}+\sqrt{\kappa\alpha_{2*}}\xi_*(t) , \quad (19)$$

$$\dot{\alpha}_2=-\gamma_2\alpha_2-\frac{g}{2}\alpha_1^2 , \quad (20)$$

$$\dot{\alpha}_{2*}=-\gamma_2\alpha_{2*}-\frac{g}{2}\alpha_{1*}^2 . \quad (21)$$

Note that the decay constant γ for the fundamental mode appears without a subscript. Noise terms $\xi(t)$ and $\xi_*(t)$ are statistically independent real Gaussian stochastic processes with zero mean and unit strength,

$$\begin{gathered}\langle\xi(t)\rangle=0=\langle\xi_*(t)\rangle , \\ \langle\xi(t)\xi(t')\rangle=\delta(t-t')=\langle\xi_*(t)\xi_*(t')\rangle .\end{gathered} \quad (22)$$

We now assume that the second-harmonic mode damps much faster than the fundamental mode so that $\gamma_2\gg\gamma$. This means that α_2 comes to equilibrium quickly and follows α_1 adiabatically. Equating $\dot{\alpha}_2=0=\dot{\alpha}_{2*}$ in Eqs. (20) and (21) and solving for the second-harmonic field amplitudes α_2 and α_{2*}, we obtain

$$\alpha_2(t)=-\frac{g}{2\gamma_2}\alpha_1^2 , \quad (23)$$

$$\alpha_{2*}(t)=-\frac{g}{2\gamma_2}\alpha_{1*}^2 . \quad (24)$$

Substituting Eqs. (23) and (24) into Eqs. (18) and (19) and introducing the scaled field amplitudes by

$$\alpha=\frac{\alpha_1}{\sqrt{n_0}},\quad \alpha_*=\frac{\alpha_{1*}}{\sqrt{n_0}},\quad \epsilon=\frac{\mathcal{E}}{\sqrt{n_0}} , \quad (25)$$

where n_0 is the threshold photon number given by

$$n_0=\frac{2\gamma\gamma_2}{g^2} , \quad (26)$$

we obtain the following equations of motion for the scaled field variables:

$$\dot{\alpha}=-\gamma(\alpha-\epsilon)-\gamma\alpha_*\alpha^2+i\sqrt{\gamma/n_0}\alpha\xi(t) , \quad (27)$$

$$\dot{\alpha}_*=-\gamma(\alpha_*-\epsilon)-\gamma\alpha\alpha_*^2+i\sqrt{\gamma/n_0}\alpha_*\xi_*(t) . \quad (28)$$

The threshold photon number n_0 is the number of photons in the fundamental mode inside the cavity at the threshold of oscillation [3]. It sets the scale of photon numbers necessary to probe the nonlinearity of the system. This number is of the order of 10^7-10^8 for typical second-harmonic generating cavities. From the equations of motion (27) and (28) for the scaled variables we can see that the noise terms are small for realistic systems. This means that we can treat the dynamics as perturbations around the deterministic steady-state obtained by ignoring the noise terms. Ignoring the noise terms in Eqs. (27) and (28) and recalling that ϵ is a real quantity we find that the deterministic steady-state amplitudes $\alpha_0=\alpha_{*0}$ are real and are determined from the relation

$$\bar{n}(1+\bar{n})^2=\epsilon^2 , \quad (29)$$

where $\bar{n}=\alpha_0^2=\alpha_{*0}^2$ is the mean photon number in units of n_0. This equation has one real positive root (physical root) for $\bar{n}$. The effect of small noise terms in Eqs. (27) and (28) is then to cause fluctuation around the steady state. Writing the field amplitudes as the sum of a coherent part and a fluctuating part,

$$\alpha=\sqrt{\bar{n}}+\delta\alpha,\quad \alpha_*=\sqrt{\bar{n}}+\delta\alpha_* , \quad (30)$$

and linearizing the equations of motion (27) and (28) we find that the fluctuations $\delta\alpha$ and $\delta\alpha_*$ obey the following coupled equations:

$$\delta\dot{\alpha}=-\gamma(1+2\bar{n})\delta\alpha-\gamma\bar{n}\delta\alpha_*+i\sqrt{\gamma\bar{n}/n_0}\xi(t) , \quad (31)$$

$$\delta\dot{\alpha}_*=-\gamma(1+2\bar{n})\delta\alpha_*-\gamma\bar{n}\delta\alpha+i\sqrt{\gamma\bar{n}/n_0}\xi_*(t) . \quad (32)$$

By introducing new variables u_1 and u_2 by

$$u_1=(\delta\alpha-\delta\alpha_*)/\sqrt{2},\quad u_2=(\delta\alpha+\delta\alpha_*)/\sqrt{2} , \quad (33)$$

we obtain two uncoupled equations

$$\dot{u}_1=\lambda_1 u_1+i\sqrt{\gamma\bar{n}/n_0}\eta_1(t) , \quad (34)$$

$$\dot{u}_2=-\lambda_2 u_2+i\sqrt{\gamma\bar{n}/n_0}\eta_2(t) , \quad (35)$$

where η_1 and η_2 are once again Gaussian white-noise processes with

$$\begin{gathered}\langle\eta_1(t)\rangle=0=\langle\eta_2(t)\rangle , \\ \langle\eta_i(t)\eta_j(t')\rangle=\delta_{ij}\delta(t-t') ,\end{gathered} \quad (36)$$

and where the decay constants λ_1 and λ_2 are given by

$$\lambda_1=\gamma(1+\bar{n}),\quad \lambda_2=\gamma(1+3\bar{n})\ . \tag{37}$$

Since the noise sources η_i are independent Gaussian random processes and Eqs. (34) and (35) are linear in u_1 and u_2, it follows that u_1 and u_2 are also independent Gaussian random processes with zero mean and with correlation functions

$$\langle u_i(t)u_j(t')\rangle=-\frac{\gamma\bar{n}}{2n_0\lambda_i}\delta_{ij}e^{-\lambda_i|t-t'|},\quad i=1,2 \tag{38}$$

in the steady state.

Field fluctuations $\delta\alpha$ and $\delta\alpha_*$ can be expressed as linear combinations of our solutions for u_1 and u_2 with the help of Eq. (33). This means that field amplitude fluctuations $\delta\alpha$ and $\delta\alpha_*$ are also Gaussian random processes with zero mean and with steady-state correlation functions given by

$$\langle\delta\alpha(\tau)\delta\alpha(0)\rangle=-\frac{\gamma\bar{n}}{4n_0}\left[\frac{e^{-\lambda_1\tau}}{\lambda_1}+\frac{e^{-\lambda_2\tau}}{\lambda_2}\right], \tag{39}$$

$$\langle\delta\alpha_*(\tau)\delta\alpha_*(0)\rangle=-\frac{\gamma\bar{n}}{4n_0}\left[\frac{e^{-\lambda_1\tau}}{\lambda_1}+\frac{e^{-\lambda_2\tau}}{\lambda_2}\right], \tag{40}$$

$$\langle\delta\alpha_*(\tau)\delta\alpha(0)\rangle=\frac{\gamma\bar{n}}{4n_0}\left[\frac{e^{-\lambda_1\tau}}{\lambda_1}-\frac{e^{-\lambda_2\tau}}{\lambda_2}\right]. \tag{41}$$

These correlation functions determine the statistical properties of $\delta\alpha$ and $\delta\alpha_*$. Higher-order correlation functions can be expressed in terms of second-order correlation functions given by Eqs. (39)–(41). From Eqs. (25), (30), (39), and (40) the time correlation matrix of the fundamental mode $\alpha_1=\alpha\sqrt{n_0}$ is given by

$$\Gamma(\tau)=n_0\bar{n}\begin{bmatrix}1&1\\1&1\end{bmatrix}-\frac{\gamma\bar{n}}{4}\begin{bmatrix}\dfrac{e^{-\lambda_1\tau}}{\lambda_1}+\dfrac{e^{-\lambda_2\tau}}{\lambda_2} & -\dfrac{e^{-\lambda_1\tau}}{\lambda_1}+\dfrac{e^{-\lambda_2\tau}}{\lambda_2}\\ -\dfrac{e^{-\lambda_1\tau}}{\lambda_1}+\dfrac{e^{-\lambda_2\tau}}{\lambda_2} & \dfrac{e^{-\lambda_1\tau}}{\lambda_1}+\dfrac{e^{-\lambda_2\tau}}{\lambda_2}\end{bmatrix}, \tag{42}$$

where we have used the definition (10) for the correlation matrix. The first matrix on the right-hand side of Eq. (42) gives the coherent Γ^0 and the second matrix gives the fluctuation part $\delta\Gamma(\tau)$ of the correlation matrix as defined in Eq. (11).

We are now ready to calculate the normalized intensity correlation function $g^{(2)}(\tau)$ of the fundamental mode. This correlation function is defined by

$$g^{(2)}(\tau)=\frac{\langle\alpha_{1*}(0)\alpha_{1*}(\tau)\alpha_1(\tau)\alpha_1(0)\rangle}{\langle\alpha_{1*}\alpha_1\rangle^2}, \tag{43}$$

and is proportional to the probability of detecting two protons in delayed coincidence. The zero delay value $g^{(2)}(0)$ indicates the probability of detecting two photons in coincidence. For a coherent beam of light $g^{(2)}(\tau)=1$ independent of the delay τ. This means for a coherent beam the correlation function factorizes, $\langle\alpha_{1*}(0)\alpha_{1*}(\tau)\alpha_1(\tau)\alpha_1(0)\rangle=\langle\alpha_{1*}\alpha_1\rangle^2$, implying that photons in a coherent beam are uncorrelated and any coincidences are purely random. The function $g^{(2)}(0)-1$ therefore measures photon coincidence probability in excess or definite of that exhibited by photons in a coherent beam. Light beams exhibiting excess coincidence probability at $\tau=0$, $g^{(2)}(0)>1$, are said to be super-Poissonian and light beams exhibiting deficit coincidence probability, $g^{(2)}(0)<1$, are said to be sub-Poissonian. On the other hand, fields for which $g^{(2)}(0)<g^{(2)}(\tau)$ are termed antibunched, while those for which $g^{(2)}(0)>g^{(2)}(\tau)$ are termed bunched. Although the properties of sub-Poissonian photon statistics and photon antibunching in general are distinct, for the simple fields that we will consider these two properties always exist together such that one implies the other. That is, $g^{(2)}(0)<1$ is sufficient to ensure both sub-Poissonian statistics and antibunching and we will therefore not maintain a distinction between the two in this paper. Both classical and quantum fields can exhibit bunching, but only quantum fields can exhibit antibunching. For this reason antibunching exhibited by photons is taken to be a signature of a quantum field. For perfect antibunching $g^{(2)}(0)=0$, which is the lower bound on $g^{(2)}(0)$. Substituting Eqs. (25) and (30) into Eq. (43), and using the Gaussian property of the fluctuations, we obtain

$$g^{(2)}(\tau)=1+\frac{2n_0\bar{n}[\delta\Gamma_{11}(\tau)+\delta\Gamma_{12}(\tau)]+[\delta\Gamma_{11}(\tau)]^2+[\delta\Gamma_{12}(\tau)]^2}{[n_0\bar{n}+\delta\Gamma_{12}(0)]^2}, \tag{44}$$

where $\delta\Gamma_{ij}(\tau)$ can be read from Eqs. (42) with the help of Eq. (11). In writing Eq. (42) we have used the symmetry properties of the correlation matrix: $\Gamma_{ij}=\Gamma_{ji}$, $\Gamma^0_{ij}=n_0\bar{n}$, $\delta\Gamma_{11}(\tau)=\delta\Gamma_{22}(\tau)$. The reason for retaining the last two terms in Eq. (44) in a linear theory is that even when fluctuations $\delta\alpha$ and $\delta\alpha_*$ are small correlation between them can be quite large. Furthermore, they serve a useful purpose by preventing an unphysical situation to arise as we shall see shortly.

For second-harmonic generation well below threshold, $\bar{n}$ is small so that $\lambda_1=\gamma(1+\bar{n})\approx\gamma$, $\lambda_2=\gamma(1+3\bar{n})\approx\gamma$, and $\lambda_2-\lambda_1=2\gamma\bar{n}$. In this limit we obtain a simple expression for $g^{(2)}(\tau)$ given by

$$g^{(2)}(\tau)=1+\frac{-4n_0e^{-\gamma\tau}+e^{-2\gamma\tau}}{4n_0^2}=\left[1-\frac{e^{-\gamma\tau}}{2n_0}\right]^2, \qquad (45)$$

which for $\tau\to 0$ leads to

$$g^{(2)}(0)=\left[1-\frac{1}{2n_0}\right]^2. \qquad (46)$$

This expression to order $1/n_0$ is the same as that obtained in Ref. [3]. The presence of higher-order terms allows us to write $g^{(2)}(0)$ as a perfect square, and this ensures that $g^{(2)}(0)$ never becomes negative. The correlation function $g^{(2)}(0)$ takes its minimum value 0 when $n_0=\frac{1}{2}$. The variation of $g^{(2)}(0)$ as a function of threshold photon number n_0 is shown in Fig. 2. As n_0 decreases $g^{(2)}(0)$ decreases until it reaches zero for $n_0=\frac{1}{2}$. It rises sharply for $n_0<\frac{1}{2}$. By contrast to Fig. 2, had we ignored the second-order terms in Eq. (44), we would have found that $g^{(2)}(0)$ falls below zero as n_0 decreases below unity, which is an unphysical situation because $g^{(2)}(0)$ has lower bound zero. By retaining the second-order terms in Eq. (44), which are of order $1/n_0^2$, we have been able to prevent this unphysical occurrence. This does not mean that the retention of second-order terms in Eq. (44) makes Eq. (46) quantitatively correct for small values of n_0. Indeed our whole analysis based upon a linearized approximation is not justified for small n_0. However, the retention of the higher-order terms offers a qualitatively useful picture of the behavior of $g^{(2)}(0)$ for small n_0, namely that of a field dominated by large fluctuations. For larger n_0 where our treatment is valid, we see from Eq. (46) that $g^{(2)}(0)$ is significantly different from unity only over a relatively small range of values of n_0. For a typical value of threshold number $n_0\approx 10^7$ we find that $g^{(2)}(0)\approx 10^{-7}$, which is an extremely small antibunching effect. With nonlinear crystals having large nonlinear coefficient of second-harmonic generation and suitable cavity designs n_0 could be lowered to perhaps $n_0\approx 10^4$. Even then, antibunching is a small effect being of order 10^{-4}.

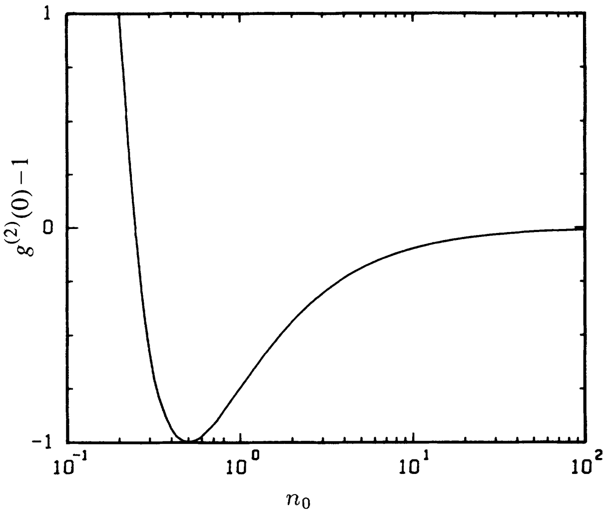

FIG. 2. Variation of $g^{(2)}(0)-1$ with the system size parameter n_0. For second-harmonic generation n_0 is the threshold photon number. For optical bistability n_0 should be replaced by n_s. Equation (44) has been used to compute the curve.

An alternative strategy for enhancing the magnitude of quantum effects typified by $g^{(2)}(0)-1$ is to use a passive cavity as a filter as sketched in Fig. 1. In order to see how this enhancement comes about let us examine Eq. (44) more carefully. We first notice that the fluctuation terms, which give rise to antibunching, are of the first order in the intensity $n_0\bar{n}$ of the coherent component. They are being compared to $n_0^2\bar{n}^2$, which is the size of the coherent contribution to $g^{(2)}(\tau)$. Thus the size of the antibunching effect is inversely proportional to the size of the coherent component. This suggests that if the coherent component can be reduced in magnitude, then the relative size of the antibunching effect might be enhanced. This conjecture is borne out by the analysis that follows. We assume that the fundamental light beam emerging from the system cavity is allowed to fall on a filter cavity as shown in Fig. 1. The spectrum of the field incident from the system cavity on the filter cavity will be given by

$$S^{\mathrm{inc}}(\Omega)=4\pi\gamma n_0\bar{n}\delta(\Omega)\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}-\gamma^2\bar{n}\begin{bmatrix}\dfrac{1}{\lambda_1^2+\Omega^2}+\dfrac{1}{\lambda_2^2+\Omega^2} & -\dfrac{1}{\lambda_1^2+\Omega^2}+\dfrac{1}{\lambda_2^2+\Omega^2}\\ -\dfrac{1}{\lambda_1^2+\Omega^2}+\dfrac{1}{\lambda_2^2+\Omega^2} & \dfrac{1}{\lambda_1^2+\Omega^2}+\dfrac{1}{\lambda_2^2+\Omega^2}\end{bmatrix}, \qquad (47)$$

where we have used Eqs. (10) and (42) and the fact that the spectrum of the field outside the system cavity is related to the spectrum of the field inside the system cavity by $S^{\mathrm{inc}}(\Omega)=2\gamma S_{\mathrm{system}}(\Omega)$ [10]. Substituting Eq. (47) into Eq. (12) we find that the time correlation matrix of the field reflected by the filter cavity will be

$$\Gamma^{\mathrm{ref}}(\tau)=2\gamma n_0\bar{n}\left[\frac{\kappa_1-\kappa_2}{\kappa_1+\kappa_2}\right]^2\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}-\frac{\gamma\bar{n}}{2}\begin{bmatrix}f_1(\tau)+f_2(\tau) & -f_1(\tau)+f_2(\tau)\\ -f_1(\tau)+f_2(\tau) & f_1(\tau)+f_2(\tau)\end{bmatrix}, \qquad (48)$$

where the function $f_i(\tau)$ is given by

$$f_i(\tau)=\frac{\gamma}{\lambda_i^2-(\kappa_1+\kappa_2)^2}\left[-\frac{\kappa_1\kappa_2}{\kappa_1+\kappa_2}e^{-(\kappa_1+\kappa_2)\tau}+\frac{\lambda_i^2-(\kappa_1-\kappa_2)^2}{\lambda_i}e^{-\lambda_i\tau}\right],\quad i=1,2\ . \qquad (49)$$

The first term on the right-hand side of Eq. (48) is the contribution from the coherent term and the second term is the contribution from the fluctuations. Following the notation introduced in Eq. (11) we shall designate these two terms by $\Gamma^{0,\mathrm{ref}}$ and $\delta\Gamma^{\mathrm{ref}}(\tau)$, respectively. For $\tau=0$, the off-diagonal terms in Eq. (48) are the mean intensity (photons/sec) of the reflected light, $\Gamma_{12}^{\mathrm{ref}}(0)\equiv I^{\mathrm{ref}}$,

$$I^{\mathrm{ref}}=2\gamma n_0\bar{n}\left[\frac{\kappa_1-\kappa_2}{\kappa_1+\kappa_2}\right]^2+\frac{\gamma\bar{n}}{2}[f_1(0)-f_2(0)] \ . \tag{50}$$

If we recall that $2\gamma n_0\bar{n}$ is the coherent part of the intensity of the light incident on the filter cavity, the factor $(\kappa_1-\kappa_2)/(\kappa_1+\kappa_2)=R$ can be identified as the amplitude reflectivity of the filter cavity when a coherent beam is incident on the cavity. This is in agreement with the classical expression for the reflectivity of a high-Q cavity [12].

The second-order intensity correlation function of the field reflected by the filter cavity is

$$g_{\mathrm{ref}}^{(2)}(\tau)=1+\frac{\langle\hat{c}_{\mathrm{ref}}^{\dagger}(0)\hat{c}_{\mathrm{ref}}^{\dagger}(\tau)\hat{c}_{\mathrm{ref}}(\tau)\hat{c}_{\mathrm{ref}}(0)\rangle-\langle\hat{c}_{\mathrm{ref}}^{\dagger}\hat{c}_{\mathrm{ref}}\rangle^2}{\langle\hat{c}_{\mathrm{ref}}^{\dagger}\hat{c}_{\mathrm{ref}}\rangle^2} \ . \tag{51}$$

Using the Gaussian property of the incident and reflected fields we can express $g_{\mathrm{ref}}^{(2)}(\tau)$ in terms of the second-order field correlation functions defined in Eqs. (48) and (49). This leads to

$$\begin{aligned} g_{\mathrm{ref}}^{(2)}(\tau)&=1+\frac{2\Gamma_{11}^{0,\mathrm{ref}}[\delta\Gamma_{11}^{\mathrm{ref}}(\tau)+\delta\Gamma_{12}^{\mathrm{ref}}(\tau)]+[\delta\Gamma_{11}^{\mathrm{ref}}(\tau)]^2+[\delta\Gamma_{12}^{\mathrm{ref}}(\tau)]^2}{[\Gamma_{12}^{0,\mathrm{ref}}+\delta\Gamma_{12}^{\mathrm{ref}}(0)]^2} \ , \\ &=1-\frac{16n_0R^2f_2(\tau)-2[f_1^2(\tau)+f_2^2(\tau)]}{[4n_0R^2+f_1(0)-f_2(0)]^2} \ , \end{aligned} \tag{52}$$

where we have used the symmetry properties $\Gamma_{ij}^{\mathrm{ref}}(\tau)=\Gamma_{ji}^{\mathrm{ref}}(\tau)$, $\Gamma_{11}^{\mathrm{ref}}(\tau)=\Gamma_{22}^{\mathrm{ref}}(\tau)$ of the correlation matrix in the present case. Expression (52) for the intensity correlation function of the reflected light can be simplified considerably if we assume that the linewidth $\kappa_1+\kappa_2$ of the filter cavity is small compared to the linewidth γ (more precisely, to the eigenvalues $\lambda_{1,2}$) of the system cavity, that is, $\kappa_1+\kappa_2\ll\gamma$. This is a reasonable approximation since our aim is to filter out the coherent component. If we further assume that the mean photon number $n_0\bar{n}$ for the fundamental field is small compared to the threshold photon number n_0, then Eq. (52) simplifies to

$$g_{\mathrm{ref}}^{(2)}(\tau)=1+\frac{-4n_0R^2e^{-\gamma\tau}+e^{-2\gamma\tau}}{(2n_0R^2+\bar{n})^2} \ . \tag{53}$$

Here in the denominator we have kept the $\bar{n}$ term because n_0R may become comparable to $\bar{n}$. A comparison of Eq. (53) with Eq. (45) shows that the filter cavity has reduced the size of coherent component compared to the contribution from fluctuations by the factor R^2. For zero delay, $\tau=0$, we obtain

$$g_{\mathrm{ref}}^{(2)}(0)=\left[1-\frac{1}{2n_0R^2}\right]^2 , \tag{54}$$

in the limit $\bar{n}\ll1$. Maximum possible antibunching $g_{\mathrm{ref}}^{(2)}(0)=0$ is achieved for the field $\hat{c}_{\mathrm{ref}}$ when the reflectivity R of the filter cavity is chosen so that

$$R_{\mathrm{opt}}=\frac{\kappa_1-\kappa_2}{\kappa_1+\kappa_2}=\frac{1}{\sqrt{2n_0}} \ . \tag{55}$$

A plot of $g_{\mathrm{ref}}^{(2)}(0)-1$ against the coherent amplitude reflectivity R, derived from Eq. (52), is shown in Fig. 3 for two different values of n_0. It will be seen that for some value R_{opt} the field reflected from the cavity exhibits almost perfect antibunching. Notice that $g_{\mathrm{ref}}^{(2)}(0)-1$ derived from Eq. (52) does not become exactly zero for R_{opt} in contrast to the prediction of Eq. (54). This is due to the finite size of the ratio $(\kappa_1+\kappa_2)/\gamma$. From Eq. (52) we can show that for $\bar{n}\ll1$ and $n_0\gg1$, $g_{\mathrm{ref}}^{(2)}(0)-1\approx\mathcal{O}[(\kappa_1+\kappa_2)/\gamma]$ when $R=R_{\mathrm{opt}}$. The value of R for maximum antibunching depends on n_0. It is also clear that cavity reflectivity must be held constant very

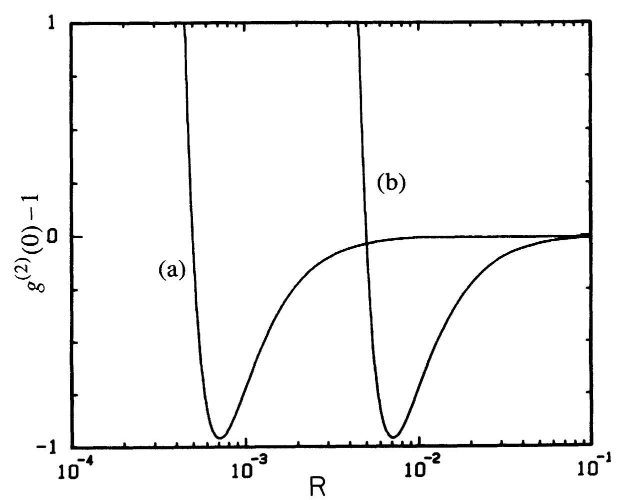

FIG. 3. Variation of $g_{\mathrm{ref}}^{(2)}(0)-1$ with the coherent amplitude reflectivity R of the filter cavity for $\kappa_1+\kappa_2=0.01\gamma$ and $\bar{n}=0.01$. Curve (a) is for $n_0=10^6$ and curve (b) is for $n_0=10^4$. Equation (25) was used to compute $g_{\mathrm{ref}}^{(2)}(0)-1$.

accurately because the minimum is rather sharp. A measure of the sensitivity of $g^{(2)}_{\rm ref}(0)-1$ to R is the full width at half maximum ΔR of the antibunching curve for which $g^{(2)}_{\rm ref}(0)-1$ stays below $-\frac{1}{2}$. This condition from Eq. (52) leads to the range of reflectivity

$$\Delta R \approx \frac{0.77}{\sqrt{n_0}} , \tag{56}$$

for which the reflected field exhibits significant antibunching. For realistic values of $n_0 \sim 10^7$ this is a very small number $\Delta R \approx 2.4\times 10^{-4}$. Thus for large values of the threshold photon number n_0 enhancement occurs in a very narrow range around $R_{\rm opt}$. For smaller values of n_0 the range is larger as can be seen from the curve corresponding to $n_0=10^4$ in Fig. 3. Note that the R axis is logarithmic. The effect of nonzero values of $\bar{n}$ is illustrated in Fig. 4, where we see that increasing values of $\bar{n}$ do not appear to significantly affect the behavior of $g^{(2)}_{\rm ref}(0)-1$ unless $\bar{n}$ becomes comparable to unity. Finally in Fig. 5 we examine the decay of the intensity correlation function in time. As we might expect, for a narrow linewidth filter cavity ($\kappa_i \ll \gamma$), the decay of correlations is monotonic and reflects the time constant γ^{-1} of the system output field. On the other hand, for a broad-linewidth filter cavity, correlations decay more rapidly than expected for the system field in the absence of filtering. Furthermore, light reflected from a broad-line filter cavity may exhibit delayed bunching, in addition to short-time antibunching. In the next section we consider a second system where a small antibunching effect can once again be enhanced by using a filter cavity.

IV. ANTIBUNCHING IN ABSORPTIVE OPTICAL BISTABILITY

Consider a single quantized mode of a high-Q cavity interacting with a collection of two-level atoms. The cavity is driven by a coherent field of amplitude $\mathcal{E}$. Since we consider only the absorptive case we shall assume that the atomic-, cavity-, and the driving-field frequencies are all the same. Then by a suitable choice of phases we can take the normalized pump-field amplitude $\mathcal{E}$ to be real. We further assume that collisional broadening is negligible so that the population decay rate $\gamma_\parallel$ and the dipole dephasing rate $\gamma_\perp$ are related by $\gamma_\parallel=2\gamma_\perp$. Using the electric dipole, rotating-wave, and Markovian approximations we can write the quantum master equation for the density matrix in the interaction picture as [5]

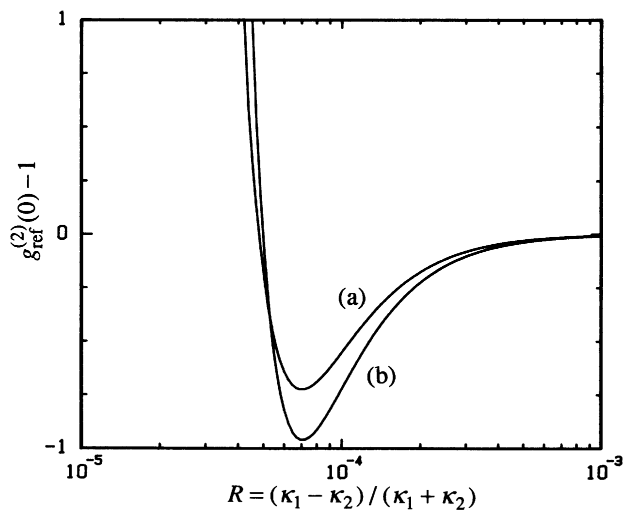

FIG. 4. Effect of $\bar{n}$ on the variation of $g^{(2)}_{\rm ref}(0)-1$ with the coherent amplitude reflectivity R of the filter cavity for $\kappa_1+\kappa_2=0.01\gamma$ and $n_0=10^8$. Curve (a) is for $\bar{n}=0.1$ and curve (b) is for $\bar{n}=0.01$. Equation (52) has been used to compute $g^{(2)}_{\rm ref}(0)-1$.

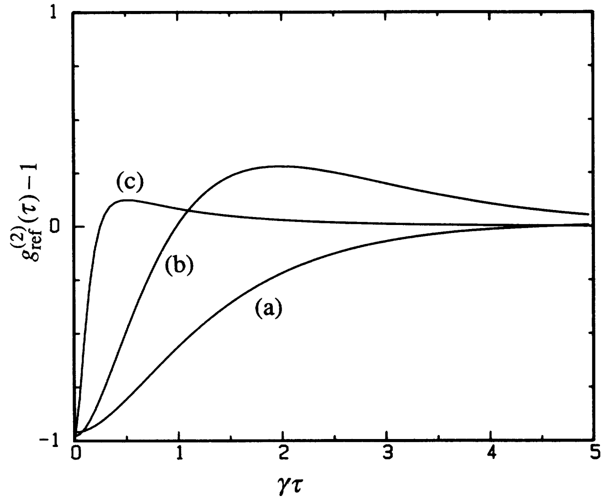

FIG. 5. Variation of the correlation function $g^{(2)}_{\rm ref}(\tau)-1$ with time for $n_0=10^8$ and $\bar{n}=0.01$. In each case R has been chosen to maximize antibunching. Curve (a) is for $\kappa_1+\kappa_2=0.01\gamma$, $R=7.16\times10^{-5}$; curve (b) is for $\kappa_1+\kappa_2=\gamma$, $R=5.05\times10^{-5}$; and curve (c) is for $\kappa_1+\kappa_2=10\gamma$, $R=2.13\times10^{-5}$. Equation (52) was used to compute $g^{(2)}_{\rm ref}(\tau)-1$. These curves also illustrate the small effect of filter cavity linewidth on the optimum value of $\kappa_1+\kappa_2$ on $g^{(2)}_{\rm ref}(0)-1$.

$$\frac{d\hat{\rho}}{dt}=-\frac{i}{\hbar}[\hat{H}_{AF},\hat{\rho}]+(\hat{\Lambda}_A+\hat{\Lambda}_F+\hat{\Lambda}_{\rm pump})\hat{\rho} , \tag{57}$$

where the interaction Hamiltonian $\hat{H}_{AF}$ is given by

$$\hat{H}_{AF}=i\hbar g(\hat{a}^\dagger\hat{J}_- - \hat{a}\hat{J}_+) . \tag{58}$$

The atomic and field dissipation and pump terms are given by

$$\hat{\Lambda}_A\hat{\rho}=\sum_{\nu=1}^{N}\gamma_\perp([\hat{\sigma}_-^{(\nu)}\hat{\rho},\sigma_+^{(\nu)}]+[\hat{\sigma}_-^{(\nu)},\hat{\rho}\hat{\sigma}_+^{(\nu)}]) , \tag{59}$$

$$\hat{\Lambda}_F\hat{\rho}=\gamma([\hat{a}\hat{\rho},\hat{a}^\dagger]+[\hat{a},\hat{\rho}\hat{a}^\dagger]) , \tag{60}$$

$$\hat{\Lambda}_{\rm pump}\hat{\rho}=\gamma\mathcal{E}([\hat{a}^\dagger,\hat{\rho}]-[\hat{a},\hat{\rho}]) . \tag{61}$$

Here $\hat{\sigma}_+^{(\nu)}$, $\hat{\sigma}_-^{(\nu)}$, and $\hat{\sigma}_3^{(\nu)}$ are Pauli atomic operators for the νth two-level atom, $\hat{a}$ and $\hat{a}^\dagger$ are the field annihilation and creation operators, γ is the cavity linewidth at the pump frequency, $g=(\mu^2\omega_0/2\hbar\epsilon_0 V_Q)^{1/2}$ is the atom-field coupling constant, μ is the transition dipole moment, and V_Q is the volume of quantization (cavity volume). Collective atomic operators $\hat{J}_\pm$ and $\hat{J}_3$ are given by

$$\hat{J}_{\pm} = \sum_{\nu=1}^{N} e^{\pm i\mathbf{k}\cdot\mathbf{r}_\nu} \hat{\sigma}_{\pm}^{(\nu)} , \tag{62}$$

$$\hat{J}_3 = \sum_{\nu=1}^{N} \sigma_3^{(\nu)} , \tag{63}$$

where we assume a traveling-wave cavity as indicated by the phase factors in $\hat{J}_{\pm}$, although our analysis can be extended to other cavity geometries [16]. The operator equation (57) for the density matrix can be converted into a c-number generalized Fokker-Planck equation [4,5]. Since we consider a large number of atoms ($N \approx 10^3 - 10^5$), this generalized Fokker-Planck equation can be truncated at the second order to obtain a Fokker-Planck-type equation. This procedure still does not lead to a Fokker-Planck equation in the usual diagonal coherent-state representation [13] because the diffusion matrix is not positive definite. This difficulty can be avoided by using positive-P representation [14]. The resulting distribution is defined on a five-dimensional complex space rather than a five-dimensional real space. This representation allows us to map quantum operators onto a set of stochastic variables according to the correspondence

$$(\hat{a}, \hat{a}^\dagger, \hat{J}_{\pm}, \hat{J}_3) \leftrightarrow (\alpha, \alpha_*, J_{\pm}, J_3) . \tag{64}$$

Here α and α_* are two independent complex variables. They are not complex conjugate of each other except in the mean and the same is true for J_+ and J_-. The equations satisfied by the c-number variables are [5,6]

$$\dot{\alpha} = -\gamma(\alpha - \mathcal{E}) + gJ_- , \tag{65}$$

$$\dot{\alpha}_* = -\gamma(\alpha_* - \mathcal{E}) + gJ_+ , \tag{66}$$

$$\dot{J}_- = g\alpha J_3 - \gamma_\perp J_- + \xi_-(t) , \tag{67}$$

$$\dot{J}_+ = g\alpha_* J_3 - \gamma_\perp J_+ + \xi_+(t) , \tag{68}$$

$$\dot{J}_3 = -\gamma_\parallel (J_3 + N) - 2g(\alpha J_+ + \alpha_* J_-) + \xi_3(t) , \tag{69}$$

where ξ_+, ξ_-, and ξ_3 are independent Gaussian white-noise processes with zero mean and correlation functions given by

$$\langle \xi_3(t)\xi_3(t')\rangle = [2\gamma_\parallel (J_3 + N) - 4g(J_+\alpha + J_-\alpha_*)]\delta(t-t') , \tag{70}$$

$$\langle \xi_-(t)\xi_-(t')\rangle = 2g\alpha J_- \delta(t-t') , \tag{71}$$

$$\langle \xi_+(t)\xi_+(t')\rangle = 2g\alpha_* J_+ \delta(t-t') . \tag{72}$$

In the good-cavity limit, $\gamma_\perp, \gamma_\parallel \gg \gamma$, the atomic variables decay much faster than the cavity field. This means that the atomic variables can be eliminated adiabatically from Eqs. (65) and (66). If we restrict ourselves to the weak-field limit, we can ignore saturation effects and write the steady-state value of the inversion $J_3 = -N$ because atoms stay essentially in the ground state. Then by setting $\dot{J}_{\pm} = 0 = \dot{J}_3$ in Eqs. (67)–(69) and solving for the atomic variables we obtain

$$J_3 \approx -N + \frac{\xi_3}{\gamma_\parallel} , \tag{73}$$

$$J_- \approx \frac{1}{\gamma_\perp}(-g\alpha N + \xi_-) , \tag{74}$$

$$J_+ \approx \frac{1}{\gamma_\perp}(-g\alpha_* N + \xi_+) . \tag{75}$$

Here we have ignored the saturation terms because at low intensities their effect is negligible. Substituting these into Eqs. (65) and (66) and introducing the scaled field variables by

$$\bar{\alpha} = \frac{\alpha}{\sqrt{n_0}}, \quad \bar{\alpha}_* = \frac{\alpha_*}{\sqrt{n_0}}, \quad \epsilon = \frac{\mathcal{E}}{\sqrt{n_0}} , \tag{76}$$

where n_0 is the saturation number given by

$$n_0 = \frac{\gamma_\perp \gamma_\parallel}{4g^2} , \tag{77}$$

we obtain the following equations for the scaled field variables:

$$\dot{\bar{\alpha}} = -\gamma(\bar{\alpha} - \epsilon) - 2C\gamma\bar{\alpha} + i\sqrt{\gamma/n_0}\,\bar{\xi}(t) , \tag{78}$$

$$\dot{\bar{\alpha}}_* = -\gamma(\bar{\alpha}_* - \epsilon) - 2C\gamma\bar{\alpha}_* + i\sqrt{\gamma/n_0}\,\bar{\xi}_*(t) . \tag{79}$$

Here the atomic cooperativity parameter $C = Ng^2/\gamma\gamma_\parallel$. The noise terms $\bar{\xi}(t)$ and $\bar{\xi}_*(t)$ in Eqs. (78) and (79) are real Gaussian white-noise processes with correlation functions

$$\langle \bar{\xi}(t)\bar{\xi}(t')\rangle = 2C\bar{\alpha}^2\delta(t-t') , \tag{80}$$

$$\langle \bar{\xi}_*(t)\bar{\xi}_*(t')\rangle = 2C\bar{\alpha}_*^2\delta(t-t') , \tag{81}$$

$$\langle \bar{\xi}(t)\bar{\xi}_*(t')\rangle = 0 . \tag{82}$$

The average in Eq. (82) is of order $\bar{\alpha}^2\bar{\alpha}_*^2$ and is negligible to the order $1/n_0$ that we are considering. In the absence of noise terms $\bar{\alpha}_* = \bar{\alpha}^*$. The deterministic (ignoring noise terms) steady-state solutions of Eqs. (78) and (79) satisfy ($\bar{\alpha}_0 = \bar{\alpha}_{*0}$ is real, $\bar{n} = \bar{\alpha}_{*0}\bar{\alpha}_0 = \bar{\alpha}_0^2 = \bar{\alpha}_{*0}^2$)

$$\bar{n} = \frac{\epsilon^2}{(1+2C)^2} . \tag{83}$$

Note that because of our choice of scaling for field variables the noise terms are small in Eqs. (78) and (79). This means that we can use a linearized description of the dynamics near the steady state described by Eq. (83). Introducing deviations $\delta\bar{\alpha}$ and $\delta\bar{\alpha}_*$ from the steady state by

$$\bar{\alpha} = \sqrt{\bar{n}} + \delta\bar{\alpha}, \quad \bar{\alpha}_* = \sqrt{\bar{n}} + \delta\bar{\alpha}_* , \tag{84}$$

with $\delta\bar{\alpha} \ll \alpha_0 = \sqrt{\bar{n}}$ and retaining only the linear terms in the deviations we obtain

$$\delta\dot{\bar{\alpha}} = -\lambda\delta\bar{\alpha} + i\sqrt{\gamma/n_0}\,\xi(t) , \tag{85}$$

$$\delta\dot{\bar{\alpha}}_* = -\lambda\delta\bar{\alpha}_* + i\sqrt{\gamma/n_0}\,\xi_*(t) . \tag{86}$$

Here the decay constant λ is given by

$$\lambda=\gamma(1+2C) , \tag{87}$$

and the noise terms $\xi(t)$ and $\xi_*(t)$ are real white-noise Gaussian processes with zero mean and correlation functions given by

$$\langle \xi(t)\xi(t')\rangle = 2C\bar{n}\gamma\delta(t-t') = \langle \xi_*(t)\xi_*(t')\rangle , \tag{88}$$

$$\langle \xi_*(t)\xi(t')\rangle = 0 . \tag{89}$$

In calculating noise correlation functions (88) and (89) we have retained only the leading terms in $\bar{n}$. Terms that have been neglected are smaller at least by a factor of $1/n_0$.

Equations (85) and (86) are linear stochastic equations driven by Gaussian white-noise processes. It follows that the variable $\delta\bar{\alpha}$ and $\delta\bar{\alpha}_*$ are Gaussian stochastic processes with zero mean and correlation functions given by

$$\langle \delta\bar{\alpha}(\tau)\delta\bar{\alpha}(0)\rangle = -\frac{C\bar{n}\gamma}{\lambda n_0}e^{-\lambda\tau} = \langle \delta\bar{\alpha}_*(\tau)\delta\bar{\alpha}_*(0)\rangle , \tag{90}$$

$$\langle \delta\bar{\alpha}_*(\tau)\delta\bar{\alpha}(0)\rangle = 0 . \tag{91}$$

Here we have retained terms to first order in $\bar{n}$. Equations (90) and (91) determine the statistical properties of $\delta\bar{\alpha}$ and $\delta\bar{\alpha}_*$ completely. Using Eq. (84) we can then write the time correlation matrix of $\alpha=\bar{\alpha}\sqrt{n_0}$ as

$$\Gamma(\tau) = n_0\bar{n}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \frac{C\bar{n}\gamma e^{-\lambda\tau}}{\lambda}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} . \tag{92}$$

Once again the first term is the coherent contribution and the second term is the contribution from fluctuations. We designate them as Γ^0 and $\delta\Gamma$ following Eq. (11). The off-diagonal elements give the mean light intensity. Since $\delta\bar{\alpha}$ and $\delta\bar{\alpha}_*$ are Gaussian processes, α and α_* are also Gaussian processes with correlation functions given by (92). Using this property, we can express the second-order intensity correlation function in terms of second-order amplitude correlation functions. This leads to an expression similar to Eq. (44). Using Eqs. (92) and (44) and retaining only leading-order terms in $\bar{n}$ we obtain

$$g^{(2)}(\tau) = 1 + \frac{-4n_s e^{-\lambda\tau} + e^{-2\lambda\tau}}{4n_s^2} , \tag{93}$$

where

$$n_s = n_0\frac{(1+2C)}{2C} . \tag{94}$$

Equation (93) agrees with previous calculations [5] except for the presence of higher-order terms that ensure that $g^{(2)}(0)$ stays positive for all values of n_0. For $\tau=0$ we obtain

$$g^{(2)}(0) = \left[1-\frac{1}{2n_s}\right]^2 . \tag{95}$$

A comparison of Eq. (95) with Eq. (46) shows that for intracavity absorption the role of parameter n_s is similar to the role played by the threshold photon number in intracavity second-harmonic generation. It is this parameter in the present case that sets the scale for photon numbers to explore the nonlinearity of the system. The maximum value of antibunching is again $g^{(2)}(0)=0$, which occurs when $n_s=n_0(1+2C)/2C=\frac{1}{2}$. The behavior of $g^{(2)}(0)$ as a function of n_s is identical to that shown in Fig. 2 with n_s replacing n_0. As n_s decreases, $g^{(2)}(0)$ decreases to zero but rises steeply for $n_s<\frac{1}{2}$. Comments similar to those made in Sec. III apply to the role played by the higher-order-fluctuation terms in our expression for $g^{(2)}(0)$ [Eq. (44)]. Without the presence of these higher-order terms in Eq. (93) the normalized intensity correlation function $g^{(2)}(0)$ becomes negative for $n_s<1$. This unphysical occurrence is prevented by the presence of these terms, which are normally neglected in treatments of antibunching in optical bistability [4,5].

Turning next to the experimental situation, we note that n_s is typically quite large in the optical domain, once again leading, as in the case of second harmonic, to small nonclassical effects. For example, in intracavity absorption experiments in Ref. [7] the saturation photon number $n_0\approx 500$ and $C\approx 10$, leading to $g^{(2)}(0)\approx 0.998$, which is less than 1% effect and would be difficult to measure. However, as in Sec. III this effect can be enhanced significantly by using a filter cavity to suppress the coherent contribution. In order to calculate the effect of the filter cavity we write down the spectrum of the field incident on the filter cavity. Using the relation $S^{\rm inc}(\Omega)=2\gamma S_{\rm system}(\Omega)$, where $S_{\rm system}(\Omega)$ is the Fourier transform of the time correlation matrix (92), we obtain

$$S_{\rm inc}(\Omega) = 4\pi\gamma n_0\bar{n}\delta(\Omega)\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \frac{4C\bar{n}\gamma^2}{\lambda^2+\Omega^2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} , \tag{96}$$

Substituting Eq. (96) into Eq. (12) and carrying out the integrals we find the time correlation matrix for the field reflected from the filter cavity is given by

$$\Gamma^{\rm ref}(\tau) = 2\gamma n_0\bar{n}R^2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - 2\gamma\bar{n}Cf(\tau)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} , \tag{97}$$

where $f(\tau)$ is given by Eq. (49) with $\lambda=\gamma(1+2C)$, and $R=(\kappa_1-\kappa_2)/(\kappa_1+\kappa_2)$ is the coherent reflectivity of the cavity. Since only one decay constant λ enters Eq. (97) we have dropped the subscript from $f(\tau)$. Following the definition introduced in Eq. (11) we denote the coherent part, represented by the first term in Eq. (91), by $\Gamma^{0,\rm ref}$ and the fluctuation part, represented by the second term in Eq. (97), by $\delta\Gamma^{\rm ref}(\tau)$. The normalized two-time intensity correlation function of the reflected field $\hat{c}_{\rm ref}$ is now readily calculated by using the Gaussian property of the incident and reflected fields. By using the symmetry properties, $\Gamma_{ij}^{0,\rm ref}=2\gamma n_0\bar{n}R^2$, $\delta\Gamma_{11}^{\rm ref}=\delta\Gamma_{22}^{\rm ref}$, and $\delta\Gamma_{12}^{\rm ref}=\delta\Gamma_{21}^{\rm ref}$ of the time correlation matrix we obtain

$$g_{\rm ref}^{(2)}(\tau)=1+\frac{2\Gamma_{11}^{0,\rm ref}[\delta\Gamma_{11}^{\rm ref}(\tau)+\delta\Gamma_{12}^{\rm ref}(\tau)]+[\delta\Gamma_{11}^{\rm ref}(\tau)]^2+[\delta\Gamma_{12}^{\rm ref}(\tau)]^2}{[\Gamma_{12}^{0,\rm ref}+\delta\Gamma_{12}^{\rm ref}(0)]^2}$$

$$=1+\frac{-2CR^2n_0f(\tau)+C^2f^2(\tau)}{(n_0R^2)^2}\ , \tag{98}$$

where in the last step we have retained only the leading order $(1/n_0)$ contributions. This amounts to dropping the contribution from $[\delta\Gamma_{12}^{\rm ref}(\tau)]^2$, which is of the order of $1/n_0^2$. Since the filter cavity is used to suppress the coherent component, we can assume $(\kappa_1+\kappa_2) \ll\gamma(1+2C)$. Using this limit we obtain a simple expression

$$g_{\rm ref}^{(2)}(\tau)=1+\frac{-4n_sR^2e^{-\lambda\tau}+e^{-2\lambda\tau}}{4(n_sR^2+\bar{n})^2} \tag{99}$$

for the intensity correlation function. The parameter n_s is given by Eq. (94). A comparison of Eq. (99) with Eq. (93) shows that the filter cavity reduces the coherent component by the factor R^2 and therefore it enhances the relative contribution from the fluctuations. For $\tau\to0$ we obtain for the zero delay correlation function [cf. Eq. (54)]

$$g_{\rm ref}^{(2)}(0)=\left[1-\frac{1}{2n_sR^2}\right]^2 \tag{100}$$

in the limit $\bar{n}\to0$. This equation predicts maximum possible antibunching $g_{\rm ref}^{(2)}(0)=0$ when R has the value

$$R_{\rm opt}=\frac{1}{\sqrt{2n_s}}=\left[\frac{C}{n_0(1+2C)}\right]^{1/2}. \tag{101}$$

Equation (101) is analogous to Eq. (55). This once again reinforces the interpretation of the saturation parameter $n_s=n_0(1+2C)/2C$ as the analog of threshold photon number for the intracavity second-harmonic generation discussed in Sec. III. The behavior of Eq. (99) as a function of R is identical to that shown in Fig. 3 with n_0 replaced by n_s. In the present case, however, we have an extra parameter, the atomic cooperativity C. The value of coherent reflectivity for maximum antibunching depends on the saturation photon number n_0 and the cooperativity parameter C. For large values of C, say, greater than 20, the dependence of $R_{\rm opt}$ on C is weak so that $R_{\rm opt}$ is essentially determined by n_0. As noted in the discussion of Fig. 4, the reflectivity R must be maintained constant very accurately because $g^{(2)}(0)-1$ is a very sensitive function of R, especially for large values of n_0. Once again a measure of this sensitivity is the range ΔR for which $g^{(2)}(0)<-0.5$. From Eq. (100) we find

$$\Delta R\approx\frac{0.77}{\sqrt{n_s}}\ . \tag{102}$$

Obviously for large values of the saturation parameter n_0 the enhancement occurs in a very narrow range around $R_{\rm opt}$. For smaller values of n_0 the range is larger as can be seen from the curve corresponding to $n_0=10^4$. The effect of nonzero values of $\bar{n}$ is similar to that shown in Fig. 4.

V. SUMMARY AND DISCUSSION

We have considered antibunching in two systems, one involving intracavity second-harmonic generation and the other involving intracavity absorption by a collection of two-level atoms. Photons emitted by both systems exhibit antibunching in time. We find that this effect for realistic systems is very small being of the order of $1/n_0$, where n_0 is some characteristic parameter that sets the scale for photon number necessary to probe the nonlinearity of the system. Since for most systems this number is large, a linearization procedure can be used to discuss quantum dynamics. We find that by retaining certain higher-order terms we can preserve the positiveness of $g^{(2)}(0)$ even for small values of the characteristic parameter n_0, although we have not explored the consistency of this procedure with respect to the original linearization and system-size expansion employed to obtain our stochastic equations in the first place. We also examine the question of enhancing the antibunching effect for realistic systems where n_0 is large. We have shown that by using a high-Q filter cavity and making measurements on the light reflected from this cavity we can achieve perfect antibunching by choosing an appropriate value of cavity reflectivity. The filter cavity removes most of the coherent component and thus enhances the relative weight of quantum noise resulting in increased antibunching. Physically this enhancement can be understood as follows. From the perspective of normally ordered operator averages the system cavity is being pumped by a coherent light beam which has a δ-function spectrum. Interaction inside the system cavity broadens the spectrum of the light that comes out of the cavity. Since the interaction is weak (n_0 large), the power contained in the broadband (incoherent) part of the spectrum is small. Antibunching, which is due to the homodyne terms between the coherent carrier and the small incoherent fluctuations, is therefore weak. The filter cavity is chosen to have a linewidth smaller than the width of the incoherent part of the spectrum so that when this cavity is tuned to the frequency of the coherent component it removes a large fraction of the coherent component. In the optimum case the reflected light contains the incoherent amplitude and the coherent carrier in roughly equal proportions so that the homodyne terms between carrier and fluctuations can result in near complete suppression of the Poisson contribution of the coherent carrier alone. Thus in the reflected light the relative weight of the incoherent component is larger than it was before filtering resulting in an enhancement of antibunching. It is interesting to note that if we utilize a filter cavity with matched mirrors so that $\kappa_1=\kappa_2$ and $R=0$, we find $g^{(2)}(0)\approx 1+1/\bar{n}^2$, which can be a large number. This means that if the coherent component is removed completely, there is no antibunching and we record the role of the fluctuations alone [the higher-order terms in Eq. (44) or (51)]. This supports the interpretation that antibunching is due to an interference between the coherent and the incoherent components. In fact, as discussed in quantitative terms by Carmichael [6], an examination of Eqs. (44), (52), (93), and (98) shows that antibunching is due to a beating of coherent and incoherent components in a fashion similar to a scheme proposed for the detection of squeezing [15]. In this sense squeezing is manifest as antibunching in both systems considered here. Experimentally, it seems possible to achieve saturation photon number of the order of 10 to 100 and threshold photon numbers of the order of 10^4-10^5.

The requirement of the linewidth of the filter cavity to obtain enhancement has to be dealt with care. There are three characteristic frequencies in the problem; they are the filter cavity linewidth $\kappa_1+\kappa_2$, filter cavity imbalance $\kappa_1-\kappa_2$, and incoherent linewidth $\sim\lambda_i$. The enhancement of antibunching depends on the relationship between these frequencies. The first limit $\lambda_i \gg \kappa_1+\kappa_2>\kappa_1-\kappa_2$ has been discussed in Sec. III and IV. In the second limit $\kappa_1+\kappa_2>\lambda_i>\kappa_1-\kappa_2$ a large fraction of the incident signal is transmitted. However as long as $(\kappa_1+\kappa_2)/\gamma<n_0$, the incoherent to coherent intensity ratio can always be tuned by $\kappa_1-\kappa_2$ to achieve almost perfect antibunching. It can be seen from Eq. (9) that in this limit, in the reflected light, low frequencies are suppressed and high frequencies are enhanced. This effectively shortens the time scale over which antibunching can be observed. Figure 5 shows the behavior of $g^{(2)}_{\rm ref}(\tau)-1$ in this limit. Note that even for $\kappa_1+\kappa_2=10\gamma$ we can have substantial antibunching. At first this result may seem surprising because one would expect that a filter cavity with a linewidth $\kappa_1+\kappa_2$ broad compared to the system cavity linewidth will transmit all the light. This, however, is not the case since as discussed above we can always find $\kappa_1-\kappa_2<\lambda_i$, which gives almost perfect antibunching. In the third limit $\kappa_1+\kappa_2>\kappa_1-\kappa_2>\lambda_i$ the filter cavity behaves like a simple reflector or a transmitter and is not of interest to us. Finally we note that the ideas discussed here have a relevance, beyond the particular examples considered, to other problems in quantum optics related to the transformation of quantum fluctuations by linear systems.

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